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A CONSIDERATION OF THE ERROR CORRECTING CODES FOR PCM RECORDING SYSTEM

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Abstract

In order to realize a higher quality PCM tape recorder, we have been insisting that an error correcting code which has a higher error correction ability must be developed. We were easily able to make an estimation of the error correction ability of codes by means of degenerating the Gilbert model into a 0-th order Markov process. We made a general-purpose simulator and proved the former estimation with it. After investigating the error correction ability and code redundancy, we discovered a new code which has an extremely high error correction ability and a low hardware cost. We named the code the 'SP code'.

1. Introduction

Since PCM tape recorders record by digitalizing the audio signals, they are capable of performing high-quality recording and playback without the signal quality at recording and playback being sacrificed. However, there are dropouts problems with the magnetic tapes, and the greatest problem has been how to correct the burst errors caused by the dropouts. In order to realize high-quality recording and playback, we proceeded with the development of an error correcting code which is ideal for PCM tape recorders.

In the past, the Gilbert model has been employed as a method of handling the burst errors theoretically. However, it is extremely difficult to use and there was no other choice than to rely on simulation with the help of a large-capacity computer. By turning the codes into blocks, we were able to degenerate the Gilbert model into a 0-th order Markov process and as a result we found that appraisal of the error correcting codes can be conducted very easily. Based on this degenerated Gilbert model, we estimated the ability of two or three codes, test-made a general-purpose hardware simulator, and followed up the theory. As a result, the estimated values and the values which were actually measured tallied to quite a considerable extent and this proved that the above theory was correct.

Furthermore, we discovered a code with an extremely high error correction ability and with comparatively low hardware costs. We discussed about this code theoretically and followed up the theory using simulator. We called this code the 'SP (sifted parity) code'.

2. Degeneration of Gilbert model

When using a video tape recorder (VTR) as a recording device for PCM signals, dropouts are produced by the dirt and scratches on the tape whose track width is a very narrow several 10 μm since the VTR performs high-density recording. Fig. 2-1 shows the dropout lengths in 20 sample tapes and also the measured frequency at which the dropouts were produced. Fig. 2-2 is a block diagram of the dropout measurement system. The dropout generating rate is extremely high at between several thousand and several hundred thousand per hour and there are also great fluctuations. Some of the longer dropouts continue for about 400 μs and the data is lost over quite long time intervals.

When the data which is recorded on the tape is played back, click noise is generated at the above frequency and it is not possible to play back music at a high quality. To counter these code errors, two methods are conceivable: error compensation using hearing characteristics and code correction using an error correcting code.

When countering the code errors, it is not possible to predict accurately the results of the countermeasure unless the nature of the dropouts is clearly established. When dropouts are produced, the data errs as burst. As a model of a channel with such burst errors the Gilbert model has been conventionally used. Fig. 2-3 shows an example of a Gilbert model. The parameters are estimated from the actually measured values given in Fig. 2-1. Fig. 2-4 shows the run distribution of the dropouts, conversely, based on these parameters. As can be clearly understood from the figure, the Gilbert model tallies to a great extent with the actually measured values at the comparatively short (< 100 μs) dropout lengths.

The Gilbert model is a 2-state Markov chain, and the state of the step before has an effect on the next state. Fig. 2-5 shows the probability for state B n steps after, supposing that the burst error is generated with a time t of 0. This is expressed by the following equation:

$$Pr = q^n + \sum_{i=1}^{n-1} i q^{i-1} Q^{n-i-1} pP \qquad (2\text{-}1)$$

The horizontal axis is expressed in time with the transmission rate for the number of steps. According to this, the probability for stage B after the generation of the burst error gradually decreases and it finally becomes a constant.

$$t_2 = \frac{p}{r + p} \qquad (2\text{-}2)$$

The mathematical analysis of this kind of Gilbert model is extremely complex and in actual fact there is no choice other than to rely on simulation using a computer.

Nevertheless, as in Fig. 2-5, it is possible to divide off the data bit lines at sufficiently long intervals from the average burst length, and then treat them as block data to convert the burst errors at bit units into random errors at block units.

In this case, the big problem is how long to make the block length. The average burst length is 11 μs and since the TV signal format must be used in order to record digital signals onto the VTR, the above relationship can be satisfied by making the 1H (63.5 μs) data one block. Fig. 2-6 shows the auto-correlation function of the block error with 1H as the block. Fig. 2-7 shows the block diagram of the equipment used to determine the auto-correlation function. This function very rapidly becomes 0 and the block error can be considered a random error.

With helical scanning VTRs it can be presumed that the dropouts which are caused by the scratches or dirt adhering to the surface of the tape can indicate the vertical correlation.

Fig. 2-8 shows the long-time auto-correlation function and it is clear that there is a strong correlation at the 245H and 490H points. Since the number of data blocks in a 1V period is 245, these indicate the vertical interval correlation. Strictly speaking then, these block errors are not random errors. Nevertheless, with ordinary codes, these errors do not stretch up to 245H, and in this range they can be treated virtually as random errors.

In this case, the Gilbert model is degenerated into a 0-th order Markov chain.

We therefore called this the degenerated Gilbert model. Fig. 2-9 gives an example of such a degenerated Gilbert model. In this model, handling the code error is very easy. In other words, the burst generation probability becomes the code error rate and it is easy to predict the results when an error correcting code is employed.

3. Error correcting codes and their ability

Quite a few error correcting codes have been announced to date but virtually all of them are difficult to apply practically due to the complexity of the hardware.

However, when it comes to multiple writing-based error correction, the circuitry can be realized quite easily. We shall now compare the composition, correction ability and simulation results of the three types of error correcting codes. The principle of error correcting based on multiple writing is as follows.

When there are two (L, R) words to be sent, the sum operation of module 2 is performed between L and R according to the equation below, and the result is taken as P:

$$P^j = L^j + R^j \pmod 2 \quad (j = 1, 2 \ldots h) \qquad (3\text{-}1)$$

(P^j, L^j and R^j are j-th bit that make up each of the words. h is the number of bits that makes up one word.)

This operation is commutative and so the following two equations are established:

$$L^j = R^j + P^j \pmod 2 \quad (j = 1, 2 \ldots h) \qquad (3\text{-}2)$$

$$R^j = P^j + L^j \pmod 2 \quad (J = 1, 2 \ldots h) \qquad (3\text{-}3)$$

Therefore, if these three words are interleaved at the recording end and recorded, and if any one word has an error, the remaining two words will correct it perfectly.

It is clear that the code can be extended in the case of n words.

If the sum operation of module 2 is performed between n words and the parity word P obtained is sent along with the data, the one word error will be completely corrected.

$$P^j = \sum_{i=1}^{n} S_i^j \pmod 2 \quad (j = 1, 2 \ldots h) \qquad (3\text{-}4)$$

$$S_k^j = P^j + \sum_{i \neq k} S_i^j \pmod 2 \quad (j = 1, 2 \ldots h) \quad (3\text{-}5)$$

The redundancy of the code decreases as n increases and the correction ability also declines. Now three codes will be considered in order of redundancy: 1.17-fold writing to create one parity from 6 words, 1.33-fold writing to create one parity from 3 words, and 1.5-fold writing to create one parity from 2 words. These are labeled in order as code A, code B and code C. Fig. 3-1 shows the method of generation for each of the codes. In the figure Li and Ri indicate the ith sample of the stereo left channel and right channel, respectively. P and Q are the parities. Each of the codes is interleaved by distance D (11Hs with codes A and B, and 10.67Hs with code C), and carried on the television sync. signal, and then recorded. During 1H period, 9 words are carried with code A, 8 words with code B and 7 words with code C, and a cyclic redundancy check character (CRCC) is added for error detection. It is not possible to detect the location of the error with CRC, and so when an error is produced, all the data on the H are considered erroneous.

Through the interleaving, all the words in a group are on different H's, and so correction is possible with a single dropout. Next, we shall consider the error correcting ability. This ability depends on the tape dropout generation rate. If the data errors on the tape are observed in H units, and if it is assumed that the number of erred H's in a sufficiently long section of 1H is r, then the tape H error rate p can be obtained by the following equation.

$$p = r/l \qquad (3\text{-}6)$$

With error correcting codes based on multiple writing, correction is not possible if at the same time the related 2Hs are erroneous. Let us now assume that 2Hs separated by interleaving distance d are erroneous in section mHs. As is clear from the auto-correlation function of Fig. 2-6 and 2-7, the 2Hs simultaneous error can be considered a random error, and the 2Hs simultaneous error rate is P^2.

The 2Hs simultaneous error H number is produced by:

$$\sum_{i=1}^{n} (m - id)P^2 \quad \text{(n: number of data words formed as a group excluding parity word)} \qquad (3\text{-}7)$$

Let's assume the expected number of data words which cannot be corrected t, when a 2Hs simultaneous error occurs, then

$$\sum_{i=1}^{n} (m - id)P^2 t \qquad (3\text{-}8)$$

data cannot be corrected in section mHs. If k data words are carried on 1H, then data error rate P_D is:

$$P_D = \frac{\sum_{i=1}^{n} (m - id)P^2 t}{mk} \doteqdot \frac{nt}{k} P^2 \quad (m \gg nd) \qquad (3\text{-}9)$$

Table 3-1 shows the results of determining the data error rates for each of the codes. Fig. 3-2 shows these values in graph form.

When the redundancy is increased the data error rate is decreased. Let's think the data error rate with code A 1, then the data error rate with code B is 1/2 and that with code C is 1/3. The data error rate for each system is about P-times compared with no correction at all. If P is assumed to be 4×10^{-4}, the improvement in the signal quality increases 2,500 times.

Next, we shall mention the results of simulating the error correcting ability of each of the codes. Fig. 3-3 shows the construction of the general-purpose code simulator which was used for the simulation. In the recorder & H error counter block, a pseudo-random pulse is generated during recording, the CRC code is added for each 1H, carried on the TV sync. signal and sent to the VTR.

During playback, the CRC is used to detect the H error, the H error periods and non-error periods are counted in H units and transferred to the minicomputer. Fig. 3-4 gives an example of the data. Head 1 indicates the error flag, and the following value is the number of erred H's. 0 indicates no errors and the following value is the number of continuous no-error H's.

The minicomputer corrects the erred words according to each error correcting codes, the data words which cannot be corrected are counted, classified, and these are sent to the printer or plotter. Fig. 3-5 gives an output example. Fig. 3-6 shows the data in graph form. If Fig. 3-2 and Fig. 3-6 are compared, it can be seen that the theoretical values and the actually measured values tally to a considerable extent.

If the redundancy is increased, the code correcting ability increases but the rate of increase is small. Even with complete 2-fold writing with a redundancy of 2, the correcting ability is improved to no more than P^2. This is because the improvement is tried within the range of the single error correction. With single error correction, the correcting ability is limited in the order of P^2. However, if two errors are corrected simultaneously, the correcting ability is of the order of P^3, and it is improved at once several thousand times. We shall mention this code in the next section.

4. SP code

The SP (sifted parity) code is composed of n data words and 2 inspection words, and any 2 words are error-corrected simultaneously. This is a code which displays a low redundancy but a high correcting ability. If the n data words are taken as Si (i = 1, 2 ...n) and the two inspection words as P and Q, then P and Q can be generated by the following equation.

$$P^j = \sum_{i=1}^{n} S_i^j \quad (\text{mod } 2) \quad (j = 1, 2 \ldots h) \tag{4-1}$$

$$Q^j = \sum_{i=1}^{n} 2^{i-1} S_i^j \quad (\text{mod } 2) \quad (j = 1, 2 \ldots h) \tag{4-2}$$

(P^j, Q^j, S_i^j indicate J-th bit of P, Q, Si.
Each of the words are composed of h bits.)

There are two independent equations, and so when the error words are two or less, the above simultaneous equations are solved, and simultaneous errors up to 2 words are all corrected.

The actual generation circuit is shown in Fig. 4-1. Interleaving is performed for each of the words, the CRC is added and they are carried on the TV signals in the same way as with the previous code. We shall now elaborate on the generation of the SP code and on the error correction operation. The explanation is undertaken with n = 6, S_1 through S_3 = L_1 through L_3, and S_4 through S_6 = R_1 through R_3.

Fig. 2 shows the P and Q generation diagram. P is a normal parity. Q is the parity with each of the data words shifted in the top direction for each bit.

The error correction is performed in the following way. It is clear that individual errors can be corrected using P or Q. Two word errors are corrected as follows. Let us assume that L_1 and R_1 are erroneous. The b_0 of L_1 is equal to the b_0 of Q, as is clear from Fig. 4-2 (b).

$$b_0\ (L_1) = b_0\ (Q) \tag{4-3}$$

Therefore, the b_0 of L_1 has been corrected. From Fig. 4-2 (a), b_0 of R_1 is a single error since b_0 of L_1 is corrected and this is easily corrected using P. Next from Fig. 4-2 (b), b_1 of L_1 is a single error since b_0 of R_1 is corrected and this is corrected using Q. In the same way, all the bits are corrected. We have assumed that L_1 and R_1 are the error words, but correction is performed in the same way even when any other data words are erroneous. Table 4-1 gives the error correcting ability of the SP code. If it is assumed at $P = 1 \times 10^{-4}$, then the improvement is about 10,000 times compared with that of code A.

Fig. 3-6 shows the simulation results. Only once was it impossible to correct the errors in six simulations. All the others were error-less. We estimate that it will be impossible to correct only once in about 100 hours.

Fig. 4-3 and 4-4 show the encoding circuit and the decoding circuit of the code. Both are extremely simple. All the words are treated in series.

5. Conclusion

We undertook our development project based on the conviction that a code displaying a high error correcting ability must be adopted in order to give PCM recording a really high quality.

In the past, there has been no choice other than to rely on simulation based on a computer in order to estimate the code ability. We, however, degenerated a Gilbert model into a 0-th order Markov process which made it possible to work out estimates using easy calculations. Furthermore, we constructed a general-purpose simulator and proved the validity of our estimates.

When using a single error correcting code, the code errors are reduced to a fraction of one in several thousand. When using the SP code which corrects 2 word errors, however, the code errors were reduced to a fraction of 1 in several tens of million compared with no correction at all at virtually the same cost as that of the hardware for realizing the single error correction code. As a result, the code errors in music reproduction can be reduced to nil.

We believe that our SP code is ideal for PCM recording.

Acknowledgements

We wish to sincerely thank Dr. Iida, director of the Wireless Research Laboratory, and Mr. Nakajima, manager, and all those who gave us their guidance and cooperation.

Reference

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(2) H. Miyagawa, Y. Iwadare, H. Imai, "Fugo-Riron", Shokodo, 1973

(3) N. Abramson, "Information Theory and Coding", McGraw-Hill Book Co., Inc. New York 1963

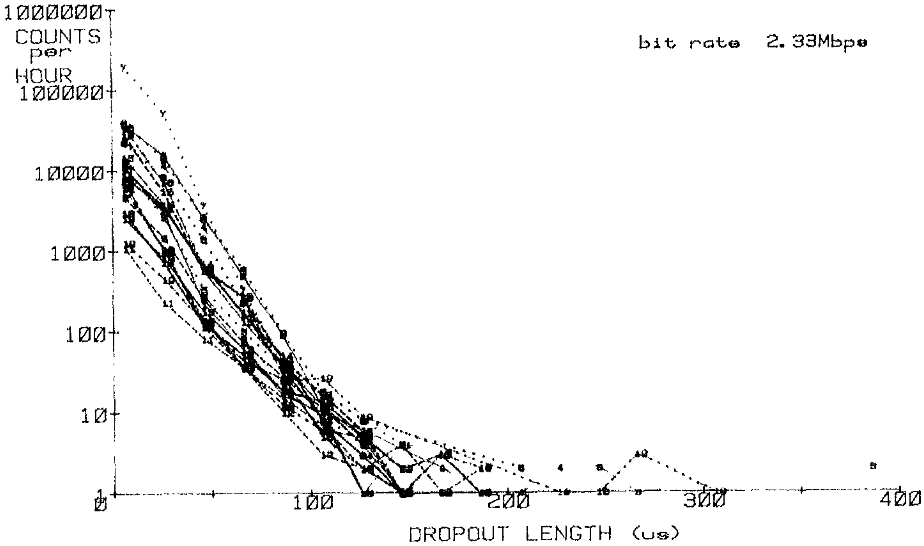


FIG 2-1 RUN DISTRIBUTION

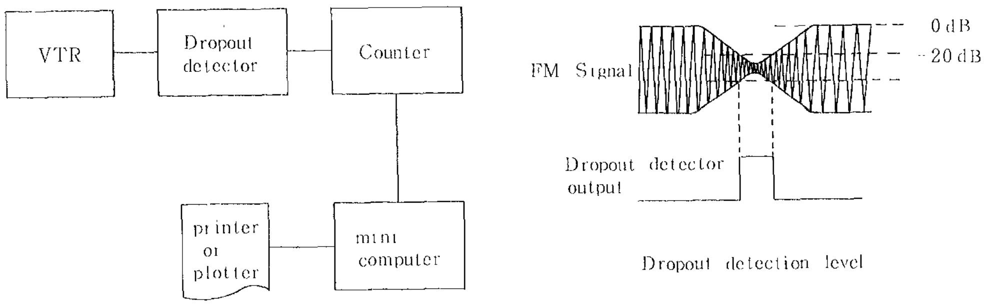


FIG 2-2 DROPOUT MEASUREMENT SCHEME

$P = 3.856 \times 10^{-6}$

$p = 3.456 \times 10^{-2}$

$t_2 = 1.116 \times 10^{-4}$

$h = 0\ 5$

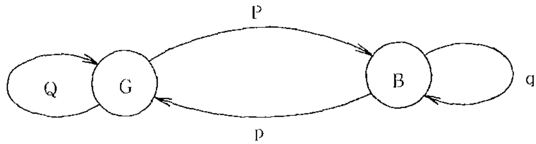


FIG2-3 GILBERT MODEL

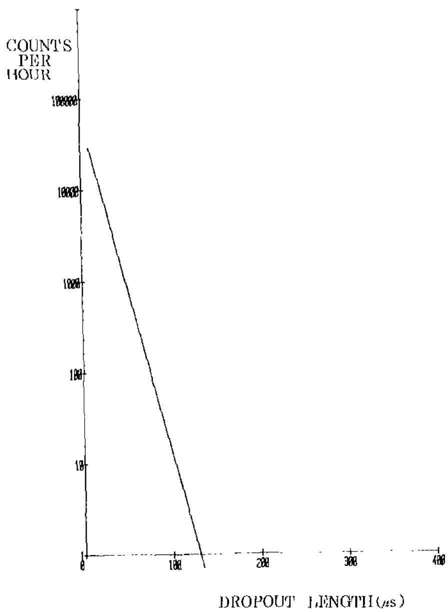


FIG2-4(a) RUN DISTRIBUTION

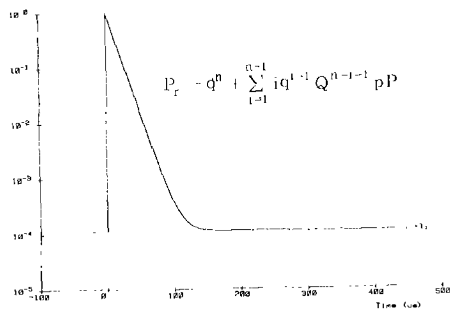


FIG2-4(b) ERROR PROBABILITY AFTER BURST

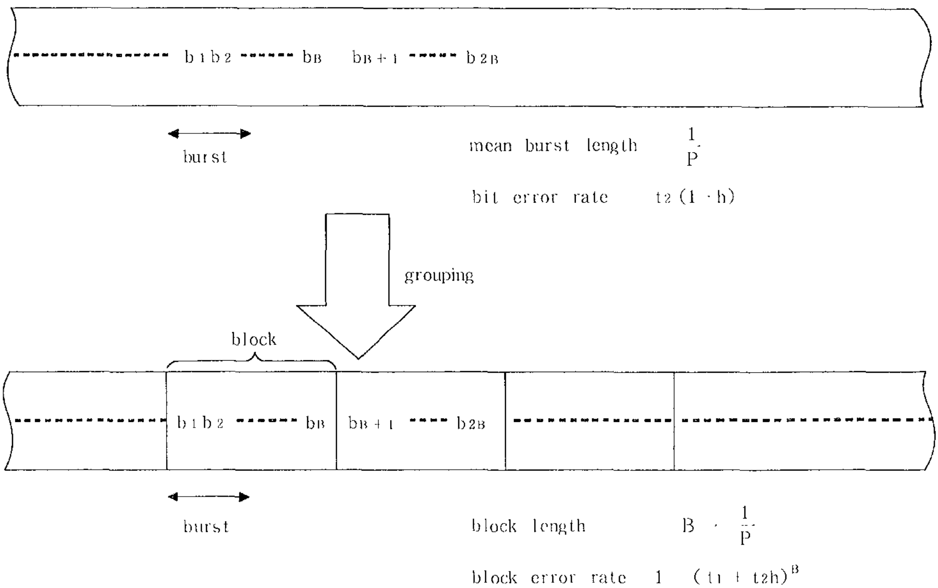


FIG 2—5 GROUPING OF CODE

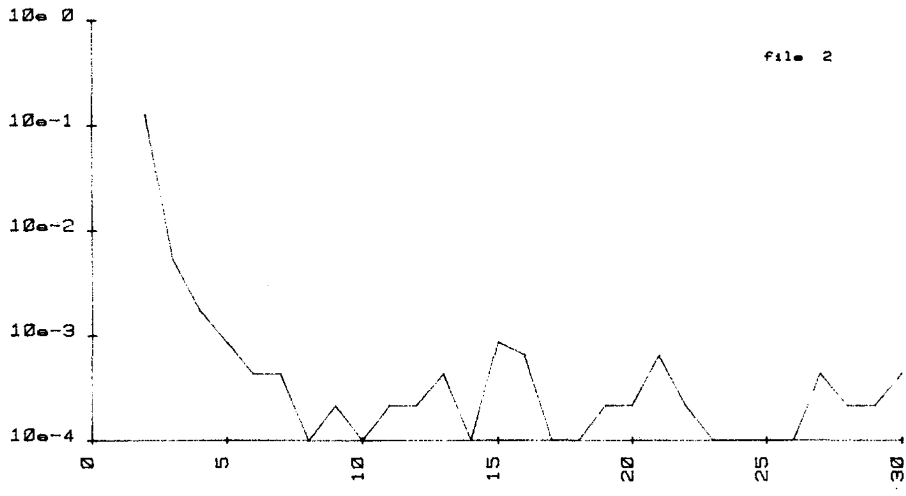
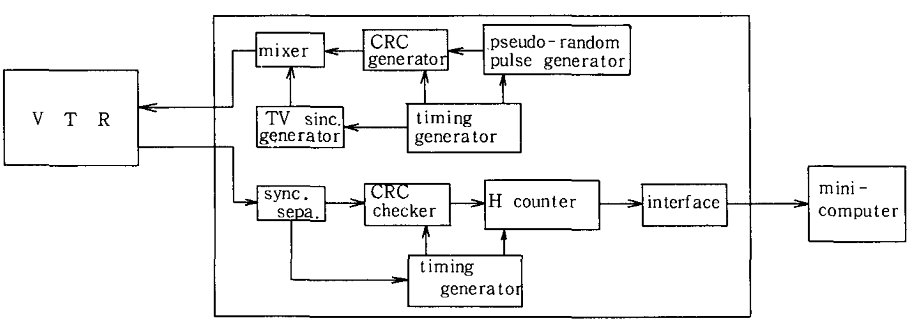
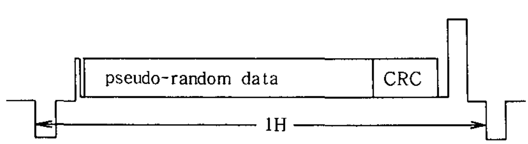


FIG 2-6 AUTOCORRELATION FUNCTION OF ERROR



(a) Autocorrelation function measuring system



(B) data format on VTR tape

FIG 2-7 AUTOCORRELATION MEASURMENT SCHEME

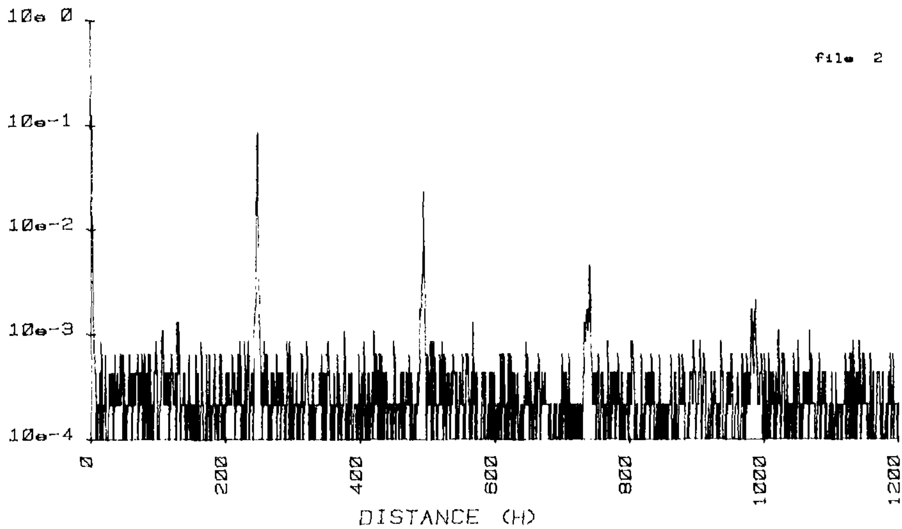
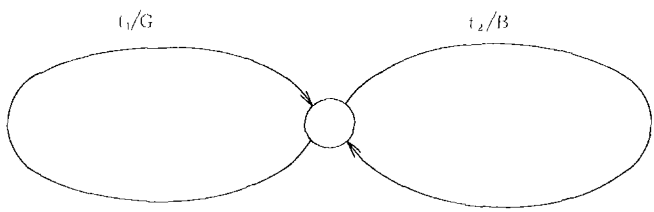


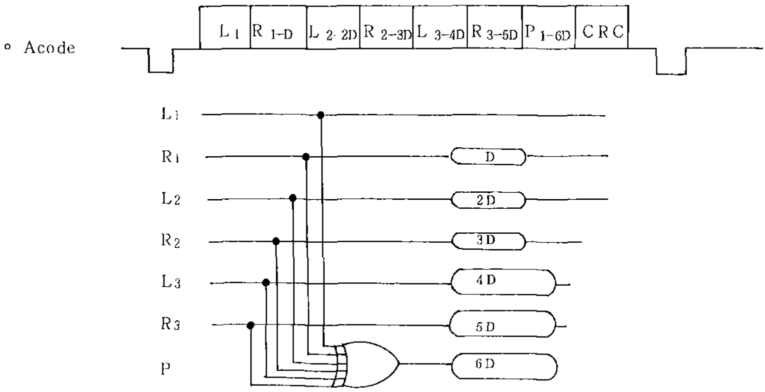
FIG 2-8 AUTOCORRELATION FUNCTION OF ERROR



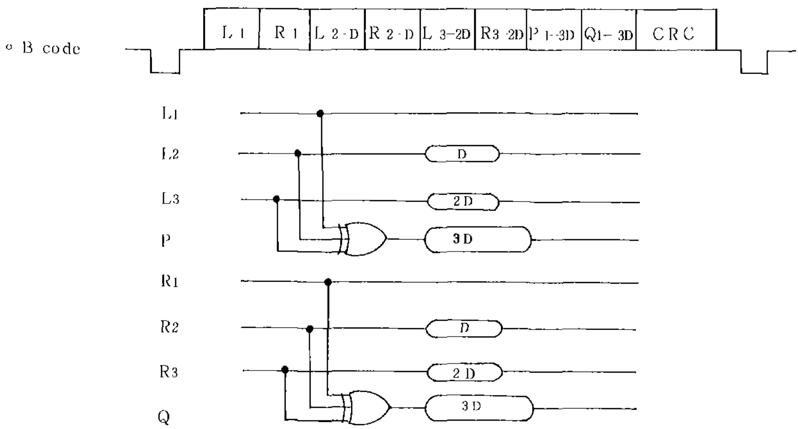
$$t_1 = \frac{P}{P + p}$$

$$t_2 = \frac{p}{P + p}$$

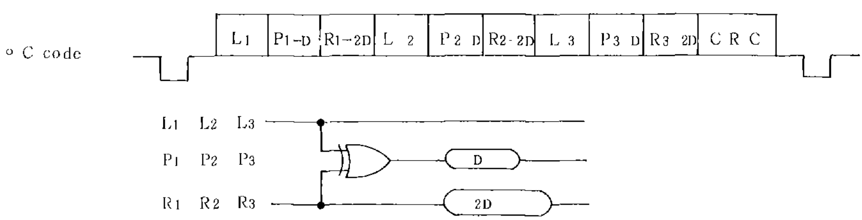
FIG2-9 DEGENERATED GILBERT MODEL



(a)



(b)



(C)

FIG3-1 EXAMPLE OF SINGLE ERROR CORRECTING CODE

TABLE3-1 ERROR CORRECTING ABILITY OF EACH CODE

item \ format	A	B	C
H error rate		P	
data error rate	$6P^2+0(P^3)$	$3P^2+0(P^3)$	$2P^2+0(P^3)$
singular error	$4.67P^2$	$1.67P^2$	$0.67P^2$
2 words continuous error	$1.33P^2$	$1.33P^2$	$1.33P^2$
3 words 〃	$0(P^3)$	$0(P^3)$	$0(P^3)$
redundancy 〃	1.17	1.33	1.5

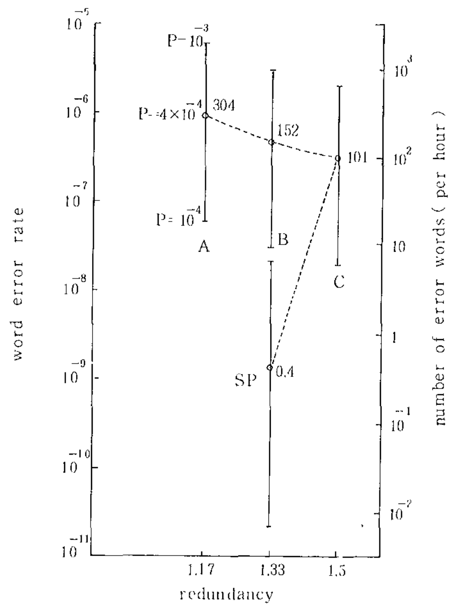


FIG3-2 ERROR CORRECTING ABILITY OF EACH CODE

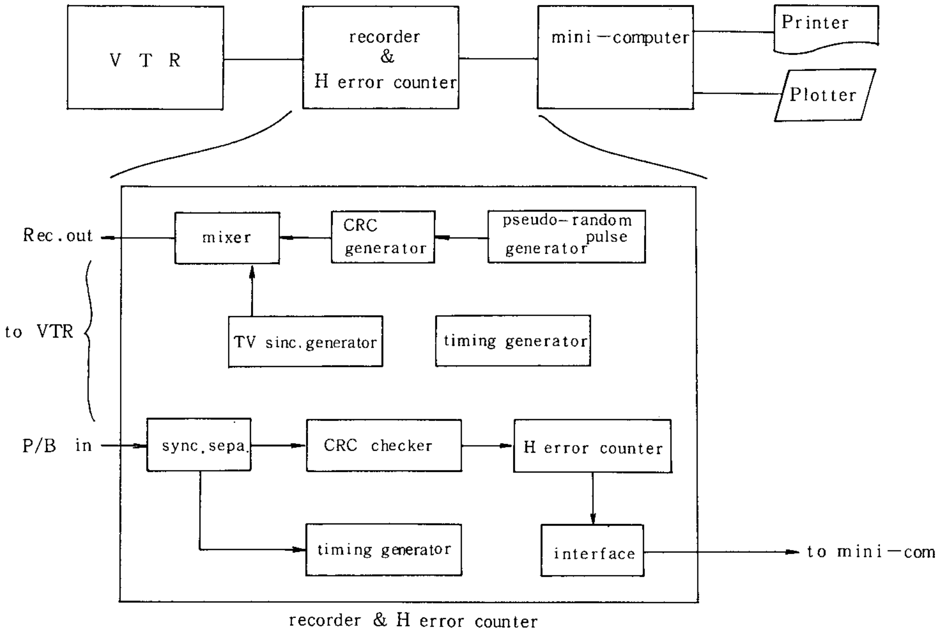


FIG 3－3 UNIVERSAL CODE SIMULATOR SCHEME

```
Data su (*1024)
8
File No.
2
***    DATA    ***
 0     70  0   6774
 1      1  0   2548
 1      1  0    246
 1      1  0    227
 1      1  0    859
 1      1  0  10815
 1      2  0   3117
 1      2  0  11546
 1      1  0   5593
 1      1  0  11766
 1      1  0   9262
 1      1  0  11559
 1      1  0   2210
 1      1  0    492
```

FIG 3－4 DATA

```
total time (min)
           21.70
total Hs
        19121289
total H errors
            4650
H error rate
       2.432E-04
total errors
              14
error rate
       1.220E-07
**distribution**
    L  channel
 single       3
 2words       2
 3words       0
    R  channel
 single       3
 2words       2
 3words       0
```

FIG 3－5 DATA

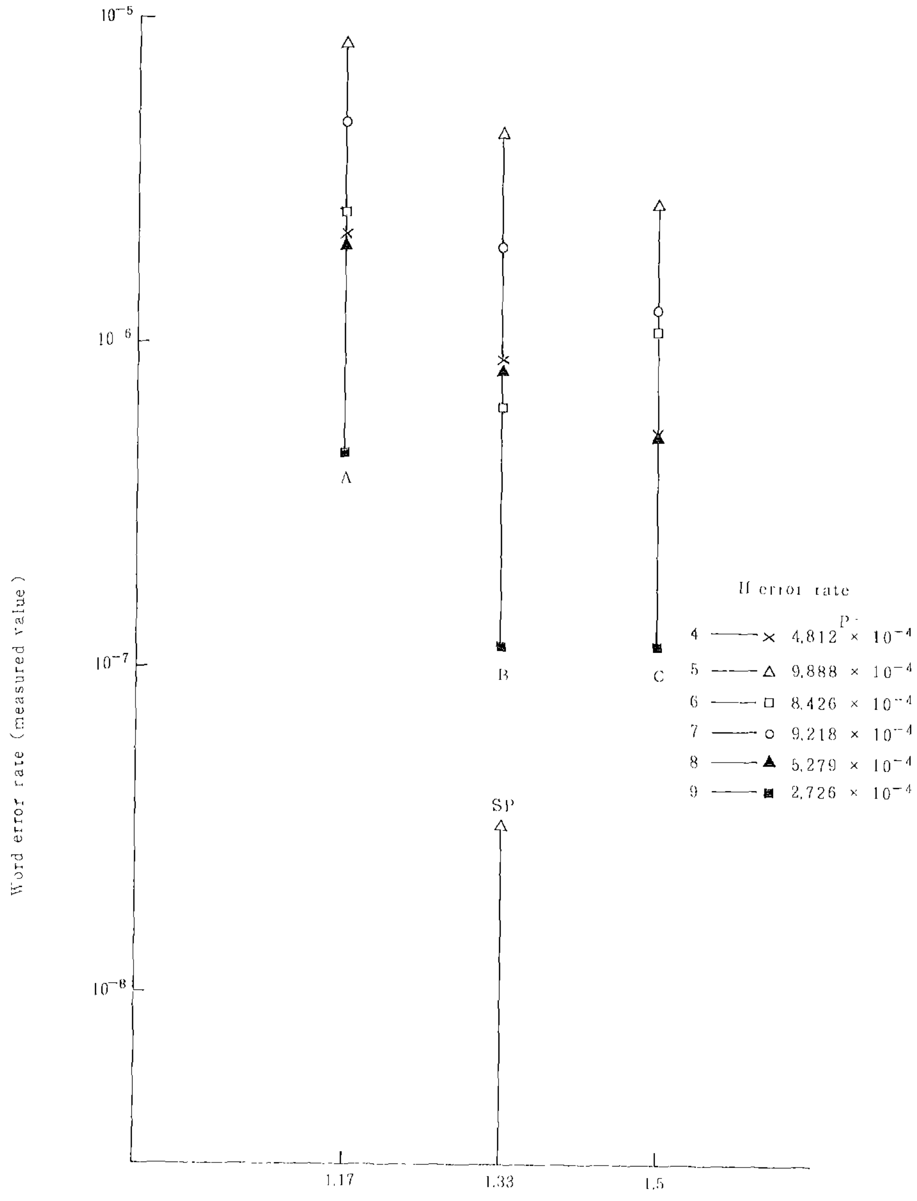


FIG3−6 ERROR CORRECTING ABILITY OF EACH CODES

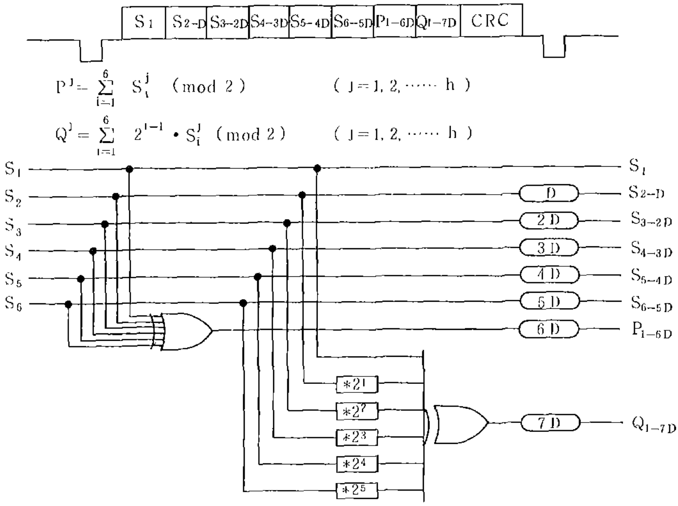


FIG4-1 SP CODE

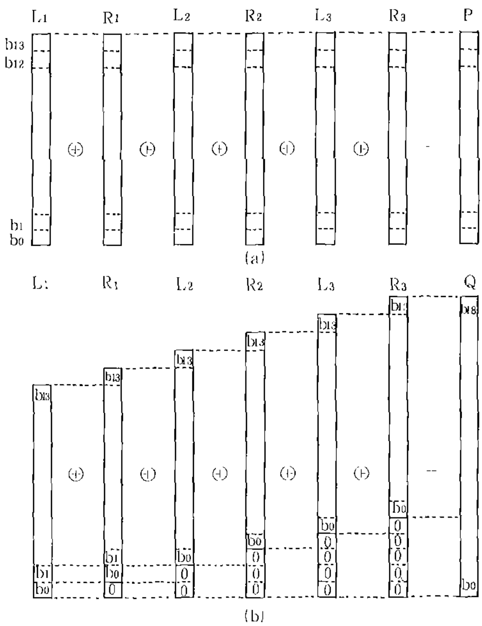


FIG4-2 CONSTRUCTION OF SP CODE

TABLE4-1 ERROR CORRECTING ABILITY OF SP CODE

item \ format	SP Code
H error rate	p
data error rate	21 p^3
singular error	13.3 p^3
2 words continuous error	6.67 p^3
3 words continuous error	p^3
redundancy	1.33

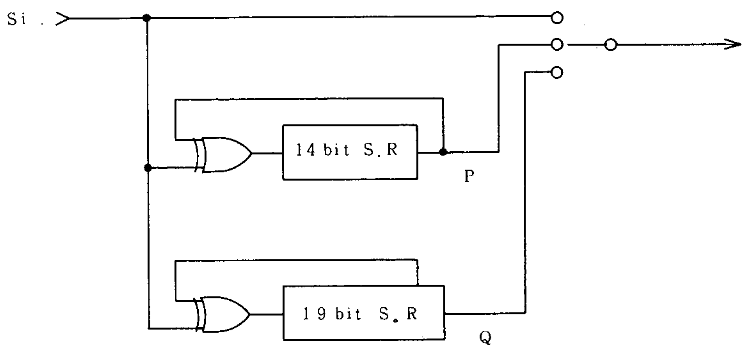


FIG4-3 ENCODER

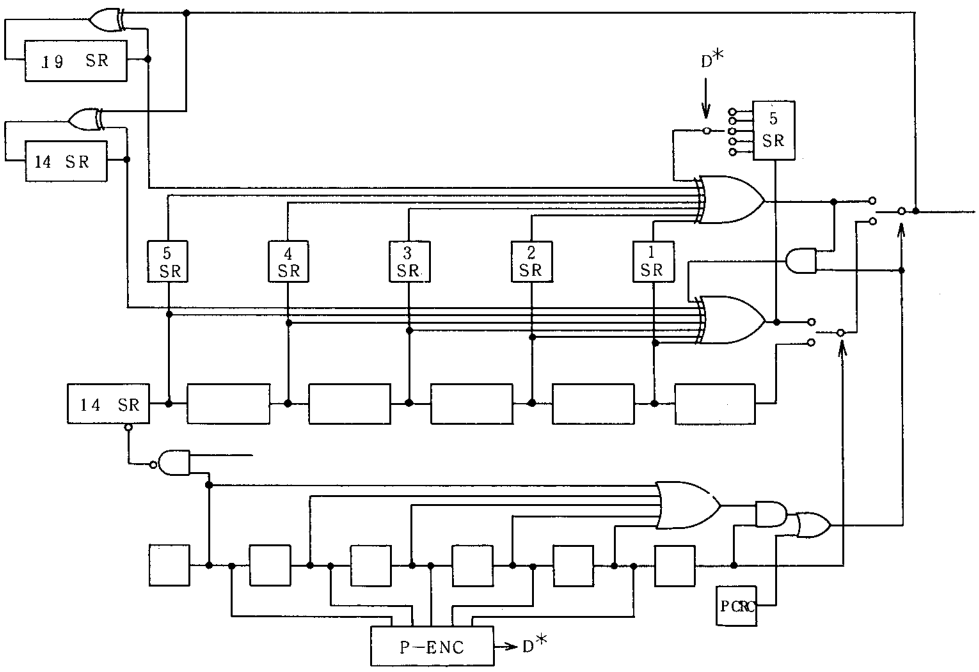


FIG4-4 DECODER